Arithmetic Progressions

2016

Very Short Answer Type Questions [1 Mark]

Question 1.
Find the 9th term from the end (towards the first term) of the A.P. 5, 9,13,185.
Solution:
Reversing the given A.P., we get
185, 181, 174, ..., 9, 5
Now, first term $(a) = 185$
Common difference, $(d) = 181 - 185 = -4$
We know that nth term of an A.P. is given by $a + (n-1)d$
Ninth term $a_9 = a + (9-1)d$
$= 185 + 8 \times (-4) = 185 - 32 = 153$

Question 2.
For what value of k will k + 9,2k -1 and 2k + 7 are the consecutive terms of an A.P.?
Solution:
Given that $k + 9$, $2k - 1$ and $2k + 7$ are in A.P.
Then $(2k-1) - (k+9) = (2k+7) - (2k-1)$
$\Rightarrow$ $k - 10 = 8 \Rightarrow k = 18$

Question 3.
For what value ofk will the consecutive terms 2k + 1, 3k + 3 and 5k -1 form an A.P.?
Solution:
Given that $2k + 1$, $3k + 3$ and $5k - 1$ are in A.P.
So, $(3k+3) - (2k+1) = (5k-1) - (3k+3)$
$\Rightarrow$ $k + 2 = 2k - 4$
$\Rightarrow$ $2k - k = 2 + 4 \Rightarrow k = 6$

Short Answer Type Questions I [2 Marks]

Question 4.

How many terms of the A.P. 18,16,14,… be taken so that their sum is zero?

Solution:

Let the number of terms taken for sum to be zero be n.

Then, sum of n terms $(S_n) = 0$

First term $(a) = 18$

Common difference $(d) = -2$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)] \Rightarrow 0 = 38 - 2n$

$\Rightarrow n = 19$

$\therefore$ Hence, sum of 19 terms is 0.

Question 5.

How many terms of the A.P. 27,24,21,… should be taken so that their sum is zero?

Solution:

In the given A.P.,

Here, first term $(a) = 27$

Common difference $(d) = -3$

Sum of n terms $(S_n) = 0$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$0 = \frac{n}{2}[2 \times 27 + (n-1)(-3)]$

$\Rightarrow 54 - 3n + 3 = 0$

$\Rightarrow 3n = 57 \Rightarrow n = 19$

Thus, the sum of 19 terms of given A.P. is zero.

Question 6.

How many terms of the A.P. 65,60, 55,… be taken so that their sum is zero?

Solution:

In the given A.P.,

First term $(a) = 65$

Common difference $(d) = 60 - 65 = -5$

Sum of n terms $(S_n) = 0$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$0 = \frac{n}{2}[2 \times 65 + (n-1)(-5)]$

$0 = 130 - 5n + 5$

$\Rightarrow -5n = -135 \Rightarrow n = 27$

$\therefore$ Hence, sum of 27 terms is zero.

Question 7.

The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

Solution:

Let a be first term and d be the common difference of the A.P. Then

$$a_n = a + (n-1)d$$

$$a_4 = a + (4-1)d$$

$$0 = a + 3d \Rightarrow a = -3d \qquad [\because \text{ Given, } a_4 = 0]$$

Now $$a_{25} = a + (25-1)d$$

$$= a + 24d = -3d + 24d = 21d = 3 \times 7d$$

Hence, $$a_{25} = 3 \times a_{11}$$

$$[\because \text{ Since } a_{11} = a + (11-1)d = -3d + 10d = 7d]$$

Question 8.

If the ratio of sum of the first m and n terms of an A.P. is m2 : n2 , show that the ratio of its mth and nth terms is (2m -1): (2n -1).

Solution:

Let S_m and S_n be the sum of first m and n terms of the A.P. Let first term and common difference of an A.P. is a and d respectively. Then

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow 2an + (mn-n)d = 2am + (mn-m)d$$

$$\Rightarrow 2a(n-m) = (mn - m - mn + n)d$$

$$\Rightarrow 2a(n-m) = (n-m)d$$

$$\Rightarrow d = 2a$$

Consider, $$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)(2a)}{a+(n-1)(2a)} = \frac{a+2am-2a}{a+2an-2a}$$

$$= \frac{2am-a}{2an-a} = \frac{2m-1}{2n-1}$$

Hence, ratio of m^{th} and n^{th} term is $2m-1 : 2n-1$.

Short Answer Type Questions II [3 Marks]

Question 9.

If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P

Solution:

Given: $$S_7 = 49, \text{ where } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$$

$$\Rightarrow 2a + 6d = 14 \Rightarrow a + 3d = 7 \qquad ...(i)$$

Similarly, $$S_{17} = 289$$

$$\frac{17}{2}[2a + (17-1)d] = 289$$

$$\Rightarrow 2a + 16d = 34 \Rightarrow a + 8d = 17 \qquad ...(ii)$$

Solving (i) and (ii), we get

$$a = 1 \text{ and } d = 2$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2 + 2n - 2] = n \times n = n^2$$

Question 10.

If the ratio of the sum of first n terms of two A.P.'s is (7n + 1): (4n + 27), find the ratio of their mth terms.

Solution:

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}$$

$$= \frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \quad ...(i)$$

Since $\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'}$ [$\because$ Let t_m, t_m be m^{th} terms of two A.P.'s]

So replacing $\frac{n-1}{2}$ by $m-1$, i.e. $n = 2m-1$ in (i)

$$\frac{t_m}{t'_m} = \frac{a+(m-1)d}{a'+(m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Thus, the ratio of their m^{th} terms is $14m - 6 : 8m + 23$.

Question 11.

The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Solution:

Let the required numbers in A.P. are $a-d, a, a+d$ respectively.

Now, $a - d + a + a + d = 15$ [$\because$ Sum of digits = 15]

$\Rightarrow 3a = 15 \Rightarrow a = 5$

According to question, number is

$100(a-d) + 10a + a + d$, i.e. $111a - 99d$

Number on reversing the digits is

$100(a+d) + 10a + a - d$, i.e. $111a + 99d$

Now, as per given condition in question,

$$(111a - 99d) - (111a + 99d) = 594$$

$$-198d = 594$$

$$d = -3$$

$\therefore$ Digits of number are $[5-(-3), 5, (5+(-3))] = 8, 5, 2$

$\therefore$ Required number is $111 \times (5) - 99(-3) = 555 + 297 = 852$

Question 12.

The sums of first n terms of three arithmetic progressions are S1, S2 and S3 respectively. The first term of each A.P. is 1 and their common differences are 1,2 and 3 respectively. Prove that S2 + S3 = 2Sr

Solution:

Here, sum of n terms of AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_1 = \frac{n}{2}[2 + (n-1)1] = \frac{n(n+1)}{2}$ [$\because$ where $a = 1, d = 1$]

$S_2 = \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}(2n) = n^2$ [$\because$ where $a = 1, d = 2$]

$S_3 = \frac{n}{2}[2 + (n-1)3]$

$= \frac{n}{2}[2 + 3n - 3] = \frac{n}{2}[3n - 1]$

Now, consider $S_1 + S_3 = \frac{n^2 + n + 3n^2 - n}{2} = \frac{4n^2}{2} = 2n^2 = 2S_2$

Question 13.

Divide 56 in four parts in A.R such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5 : 6.

Solution:

Let the four parts of the A.P. are $a-3d, a-d, a+d, a+3d$

Now, $a-3d+a-d+a+d+a+3d = 56$ [$\because$ Sum = 56]

$$\Rightarrow \quad 4a = 56 \Rightarrow a = 14$$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6} \quad [\because \text{ Putting } a = 14]$$

$$\Rightarrow \quad \frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

$$\Rightarrow \quad 1176-54d^2 = 980-5d^2$$

$$\Rightarrow \quad 49d^2 = 196 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

Thus, 4 parts are $a-3d, a-d, a+d, a+3d$, i.e. 8, 12, 16, 20.

Question 14.

The pih, 9th and rth terms of an A.P. are a, b and c respectively. Show that a(q – r) + b(r-p) + c(p – q) = 0

Solution:

Let A and d be the first term and common difference of the given A.P., then

$$a_p = A + (p-1)d = a \quad \text{...(i)}$$

$$a_q = A + (q-1)d = b \quad \text{...(ii)}$$

$$a_r = A + (r-1)d = c \quad \text{...(iii)}$$

Now, subtracting (*i*) and (*ii*), we get

$$(p-q)d = a-b$$

$$p-q = \frac{a}{d}-\frac{b}{d}$$

Multiplying by '*c*' both sides,

$$c(p-q) = \frac{ca}{d}-\frac{cb}{d} \quad \text{...(iv)}$$

Now, (*ii*) – (*iii*), we get

$$(q-r)d = b-c$$

$$q-r = \frac{b}{d}-\frac{c}{d}$$

Multiplying by '*a*' both sides,

$$a(q-r) = \frac{ab}{d}-\frac{ac}{d} \quad \text{...(v)}$$

Now, (*iii*) – (*i*), we get

$$(r-p)d = c-a$$

$$(r-p) = \frac{c}{d}-\frac{a}{d}$$

Multiplying by '*b*' both sides,

$$(r-p)b = \frac{bc}{d}-\frac{ba}{d} \quad \text{...(vi)}$$

Adding (*iv*), (*v*) and (*vi*), we get

$$a(q-r)+b(r-p)+c(p-q) = \frac{ab}{d}-\frac{ac}{d}+\frac{bc}{d}-\frac{ba}{d}+\frac{ca}{d}-\frac{cb}{d} = 0$$

Question 15.

The sums of first n terms of three A.Ps' are S1, S2 and S3. The first term of each is 5 and their common differences are 2,4 and 6 respectively. Prove that S1 + S3 = 2Sr

Solution:

Here $a = 5$ and $d_1 = 2, d_2 = 4$ and $d_3 = 6$. Let sum of 'n' terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Now,

$$S_1 = \frac{n}{2}[2 \times 5 + (n-1)2]$$

$$= \frac{n}{2}[10 + 2n - 2] = \frac{(2n+8)n}{2} = n(n+4)$$

$$S_2 = \frac{n}{2}[2 \times 5 + (n-1)4]$$

$$= \frac{n}{2}[10 + 4n - 4] = \frac{n(4n+6)}{2} = n(2n+3) = 2n^2 + 3n$$

$$S_3 = \frac{n}{2}[2 \times 5 + (n-1)6]$$

$$= \frac{n}{2}[10 + 6n - 6] = \frac{n}{2}[6n + 4] = n(3n+2)$$

Consider

$$S_1 + S_3 = n^2 + 4n + 3n^2 + 2n = 4n^2 + 6n = 2(2n^2 + 3n) = 2S_2$$

Question 16.

A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Solution:

Let total time be $(n-1)$ minutes in which the police catch the thief.

Since thief ran 1 minute before police start running.

$\because$ Time taken by thief before he was caught $= (n - 1 + 1) = n$ minute

Then total distance covered by thief $= (100 \times n)$ metres

Total distance covered by policeman in $(n-1)$ minute

$$= 100 + 110 + 120 + ... + (n-1) \text{ terms}$$

$$= \frac{(n-1)}{2}[2000 + (n-2)10] \quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

According to question,

Total distance covered by thief in 'n' minute

$$= \text{total distance covered by policeman in } (n-1) \text{ minute}$$

$$100n = \frac{(n-1)}{2}[200 + (n-2)10]$$

$$\Rightarrow 200n = (n-1)[200 + 10n - 20]$$

$$\Rightarrow 200n = (n-1)(10n + 180)$$

$$\Rightarrow 200n = 10n^2 + 180n - 10n - 180$$

$$\Rightarrow 10n^2 - 30n - 180 = 0$$

$$\Rightarrow n^2 - 3n - 18 = 0 \Rightarrow n^2 - 6n + 3n - 18 = 0$$

$$\Rightarrow n(n-6) + 3(n-6) = 0 \Rightarrow (n-6)(n+3) = 0$$

$$\Rightarrow n = 6 \text{ or } n = -3 \text{ (rejected)}$$

Hence, time taken by policeman to catch the thief is $(6 - 1)$, i.e. 5 minutes.

Question 17.

A thief, after committing a theft, runs at a uniform speed of 50 m/minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every succeeding minute. After how many minutes, the policeman will catch the thief?

Solution:

Suppose policeman catches thief after t minutes.
Given: uniform speed of thief = 50 m/min.
Since thief ran 2 minutes before police start running,
$\therefore$ Distance covered by thief in $(t + 2)$ minutes

$$= 50 \text{ m/min} \times (t + 2) \text{ min} = 50(t + 2) \text{ m}$$

An AP is formed in case of the policeman, i.e. 60, 65, 70, ...
$\therefore$ Distance covered by policeman in t minutes

$$= \frac{t}{2}[2 \times 60 + (t - 1) \times 5] = 60t + \frac{5t}{2}(t - 1)$$

Now, when policeman catches the thief, we have

$$\Rightarrow \quad 60t + \frac{5t^2}{2} - \frac{5t}{2} = 50t + 100 \Rightarrow t^2 + 3t - 40 = 0$$

$$\Rightarrow \quad (t + 8)(t - 5) = 0$$

$$\Rightarrow \quad t + 8 = 0 \quad \text{or} \quad t - 5 = 0$$

$$\Rightarrow \quad t = -8 \quad \text{or} \quad t = 5$$

$$\therefore \quad t = 5, \text{ since } t \text{ cannot be negative.}$$

Thus, the policeman catches the thief after 5 minutes.

Question 18.

The sum of three numbers in A.P. is 12 and sum of their cubes is 288, Find the numbers.

Solution:

Let the three numbers in A.P. are $a - d, a, a + d$

Then $a - d + a + a + d = 12$ $\quad$ [$\because$ Given that, $S_3 = 12$]

$$\Rightarrow \quad 3a = 12 \Rightarrow a = 4$$

Also, $(a - d)^3 + a^3 + (a + d)^3 = 288$ $\quad$ [$\because$ Sum of their cubes = 288]

$$\Rightarrow \quad (4 - d)^3 + (4)^3 + (4 + d)^3 = 288$$

$$\Rightarrow \quad 64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$$

$$\Rightarrow \quad 24d^2 + 192 = 288 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

For $d = 2$, the numbers will be 2, 4, 6. For $d = -2$, numbers will be 6, 4, 2.
Hence, required numbers are 2, 4, 6.

Question 19.

The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceding the house numbered X is equal to sum of the numbers of houses following X.

Solution:

The A.P. of numbers of houses preceding house numbered x is: $1 + 2 + 3 + ... + (x - 1)$

$\therefore$ Sum, $S_n = \frac{n}{2}[2a + (n-1)d]$, where $a \to$ first term, $d \to$ common difference

$$= \frac{(x-1)}{2}[2 \times 1 + (x - 1 - 1) \times 1]$$

$$= \frac{(x-1)}{2} \times [2 + x - 2] = \frac{x(x-1)}{2}$$

Now, A.P. of total number of houses following x is: $(x + 1) + (x + 2) + ... + 49$

$$n = 49 - (x + 1) + 1 = 49 - x$$

$\therefore$ Sum of these numbers, $S_n = \frac{n}{2}[a + l]$, where l is last term

$$= \frac{(49-x)}{2}[x + 1 + 49] = \frac{(49-x)}{2}(x + 50)$$

According to question,

$$\frac{x(x-1)}{2} = \frac{(49-x)(x+50)}{2}$$

$\Rightarrow$ $x^2 - x = 49x + 2450 - x^2 - 50x$

$\Rightarrow$ $2x^2 = 2450$

$\Rightarrow$ $x^2 = 1225 \Rightarrow x = 35$

Justification:

Now, A.P. of numbers before house numbered $x = 1 + 2 + ... + 34$

$\therefore$ $S_{34} = \frac{34}{2}[a + l] = \frac{34}{2} \times [1 + 34] = 17 \times 35 = 595$

Now, A.P. of numbers following house numbered $x = 36 + 37 ... + 49$

$\therefore$ $S' = \frac{14}{2}[36 + 49] = 7 \times 85 = 595$

Hence, for value of $x = 35$, the sum of numbers of houses preceding house numbered x is equal to sum of numbers of houses following x.

Question 20.

Reshma wanted to save at least ? 6,500 for sending her daughter to school next year (after 12 months). She saved ? 450 in the first month and raised her savings by ? 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?

Solution:

The amounts saved form an A.P. 450, 470, 490, ... in which

first term (a) = ₹ 450

Common difference (d) = ₹ 20

Total terms (n) = 12 (number of months)

Then, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{12} = \frac{12}{2}[2 \times 450 + (12 - 1)(20)] = 6[900 + 220] = 6 \times 1120 = 6720$$

Now, $6720 > 6500$

$\therefore$ Reshma will be able to send her daughter to school as she has saved more than ₹ 6500.

Now, Reshma is very much concerned about her daughter's education. She is awared and dedicated towards her daughter is education.

2015

Very Short Answer Type Question [1 Mark]

Question 21.

Find the 25th term of the A.P. – 5, -5/2, 0, 5/2……………

Solution:

We have, first term $(a) = -5$, second term $(a_2) = \frac{-5}{2}$, third term $(a_3) = 0$

$$d = \frac{-5}{2} - (-5) = \frac{5}{2}$$

Now, we know that $a_n = a + (n-1)d$

$$a_{25} = a + 24d = -5 + 24 \times \frac{5}{2} = 55$$

Short Answer Type Questions I [2 Marks]

Question 22.

Find the middle term of the AP 6,13,20,…, 216.

Solution:

Given: AP is 6, 13, 20, ..., 216

Here first term, $a = 6$; common difference, $d = 13 - 6 = 7$, n^{th} term, $a_n = 216$

$\Rightarrow \quad a + (n-1)d = 216 \Rightarrow 6 + 7(n-1) = 216 \Rightarrow 7n = 217 \Rightarrow n = 31$

Since, the number of terms in AP are 31, so, the middle most term is 16^{th} term.

$$\left[\because \text{ middle term} = \frac{(31+1)}{2} = 16^{th} \text{ term}\right]$$

$\therefore \quad 16^{th}$ term, $a_{16} = a + 15d = 6 + 15 \times 7 = 111.$

$\Rightarrow \quad 213 - 8(n-1) = 37 \Rightarrow 213 - 8n + 8 = 37$

$\Rightarrow \quad 8n = 221 - 37 \Rightarrow 8n = 184 \Rightarrow n = 23$

Since the number of terms in AP are 23, so, the middle most term is 12^{th} term.

$$\left[\because \text{ middle term} = \frac{(23+1)}{2} = 12^{th} \text{ term}\right]$$

$\therefore \quad a_{12} = a + 11d = 213 + 11(-8) = 125.$

Question 23.

Find the middle term of the AP 213,205,197,…, 37.

Solution:

Given AP is 213, 205, 197, ... , 37.

Here, first term, $a = 213$; common difference, $d = 205 - 213 = -8$, n^{th} term, $a_n = 37$.

$\Rightarrow \quad a + (n-1)d = 37$

Question 24.

In an AP, if S5 + S7 = 167 and S10 = 235, then find the AP, where Sn denotes the sum of its first n terms.

Solution:

Let 1st term of the AP $= a$ and common difference $= d$

Now $S_5 + S_7 = 167$

$\Rightarrow \quad \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167 \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\Rightarrow \quad 5a + 10d + 7a + 21d = 167 \Rightarrow 12a + 31d = 167 \qquad ...(i)$

Also, $S_{10} = 235 \Rightarrow \frac{10}{2}(2a + 9d) = 235 \Rightarrow 2a + 9d = 47 \quad ...(ii)$

Multiplying equation (*ii*) by 6, we get

$\Rightarrow \quad 6(2a + 9d) = 6 \times 47 \Rightarrow 12a + 54d = 282 \qquad ...(iii)$

$\therefore$ Subtracting equation (*i*) from (*iii*), to get

$$\begin{aligned} 12a + 54d &= 282 \\ -12a - 31d &= -167 \\ \hline 23d &= 115 \end{aligned}$$

$\therefore \quad d = 5$

Putting 'd' in (*ii*) equation, $a = 1$

$\therefore$ Required AP is 1, 6, 11, ...

Question 25.

The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference

Solution:

Let 1st term of the AP $= a$

Common Difference $= d$

Now, $a_4 = 11$ [Given]

$\Rightarrow a + 3d = 11 \Rightarrow a = 11 - 3d$ ...(i)

Also, $a_5 + a_7 = 34$ [Given]

$a + 4d + a + 6d = 34$

$2a + 10d = 34 \Rightarrow a = 17 - 5d$ ...(ii)

From (i) and (ii) $11 - 3d = 17 - 5d$

$\Rightarrow 2d = 6 \Rightarrow d = 3$

Question 26.

The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference of the A.P.

Solution:

Let Ist term of the AP $= a$ and Common difference $= d$

Now, $a_5 = 20 \Rightarrow a + 4d = 20 \Rightarrow a = 20 - 4d$ ...(i)

Also $a_7 + a_{11} = 64$ [Given]

$a + 6d + a + 10d = 64 \Rightarrow 2a + 16d = 64$

$a + 8d = 32$

$\Rightarrow 20 - 4d + 8d = 32$ [using equation (i)]

$4d = 12 \Rightarrow d = 3$

Question 27.

The ninth term of an A.P is -32, and the sum of eleventh and thirteenth terms is -94.find the common difference of the A.P

Solution:

Let Ist term of the AP $= a$ and Common difference $= d$

Now, $a_9 = -32$ [Given]

$\Rightarrow a + 8d = -32 \Rightarrow a = -32 - 8d$ ...(i)

Also, $a_{11} + a_{13} = -94$ [Given]

$a + 10d + a + 12d = -94 \Rightarrow 2a + 22d = -94$

$a + 11d = -47 \Rightarrow -32 - 8d + 11d = -47$ [$\because$ using equation (i)]

$\Rightarrow 3d = -15 \Rightarrow d = -5$

Short Answer Type Questions

Question 28.

If the sum of the first n-terms of an AP is $\frac{1}{2}(3n^2 + 7n)$, then find its n^{th} term. Hence write its 20^{th} term. [Delhi]

Solution:

Given, sum of first n-term $S_n = \frac{3}{2}n^2 + \frac{7}{2}n$

So, $a_1 = S_1 = \frac{3}{2}(1^2) + \frac{7}{2}(1) = \frac{3}{2} + \frac{7}{2} = 5$

Now, $S_2 = \frac{3}{2}(2)^2 + \frac{7}{2} \times 2 = 13$

Then $a_2 = S_2 - S_1 = 13 - 5 = 8$

Common difference $d = a_2 - a_1 = 8 - 5 = 3$

Now, n^{th} term, $a_n = a_1 + (n-1)d = 5 + 3(n-1) = 3n + 2$

$\therefore$ 20^{th} term, $a_{20} = a_1 + 19d = 5 + 19 \times 3 = 62.$

Question 29.

If S_n denotes the sum of first n-terms of an AP, prove that $S_{30} = 3[S_{20} - S_{10}]$.

Solution:

Consider

$$\begin{aligned} \text{RHS} &= 3(S_{20} - S_{10}) \\ &= 3\left[\frac{20}{2}(2a + 19d) - \frac{10}{2}(2a + 9d)\right] \quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\} \\ &= 3[10(2a + 19d) - 5(2a + 9d)] \\ &= 3(20a + 190d - 10a - 45d) \\ &= 3(10a + 145d) = 3 \times 5(2a + 29d) \\ &= \frac{30}{2}(2a + 29d) = S_{30} = \text{LHS} \end{aligned}$$

Hence, proved.

Question 30.

It Sn, denotes the sum of first n-terms of an AP. Prove that: S12 = 3 (S8 – S4)

Solution:

Let 'a' be first term, 'd' be common difference of given AP.

Consider

$$\begin{aligned} \text{RHS} &= 3(S_8 - S_4) \\ &= 3\left[\frac{8}{2}(2a + 7d) - \frac{4}{2}(2a + 3d)\right] \quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\} \\ &= 3[4(2a + 7d) - 2(2a + 3d)] \\ &= 3(4a + 22d) = 3 \times 2(2a + 11d) \\ &= \frac{12}{2}(2a + 11d) = S_{12} = \text{LHS} \end{aligned}$$

Hence, proved.

Question 31.

The 14th term of an AP is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms.

Solution:

Let 1st term of AP $= a$ and common difference $= d$

A.T.Q. $a_{14} = 2a_8$

$\Rightarrow$ $a + 13d = 2(a + 7d) \Rightarrow a = -d$

Also, given $a_6 = -8 \Rightarrow a + 5d = -8$

$\Rightarrow$ $-d + 5d = -8 \Rightarrow d = -2$

$\Rightarrow$ $a = 2$

$\therefore$ Sum of first 20 terms, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times -2) = 10 \times (-34) = -340$

$\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

Question 32.

The 16th term of an AP is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms.

Solution:

Let 3^{rd} term $= a_3 = a + 2d$

16^{th} term $= a_{16} = a + 15d$

Let 1^{st} term of the AP = a and Common difference = d

A.T.Q., $a_{16} = 5 \times a_3$ [Given]

$\Rightarrow$ $a + 15d = 5(a + 2d) \Rightarrow a + 15d = 5a + 10d$

$5d = 4a \Rightarrow a = \frac{5}{4}d$ ...(i)

Also, given $a_{10} = 41 \Rightarrow a + 9d = 41$

$\Rightarrow$ $\frac{5}{4}d + 9d = 41$ [Using eq. (i)]

$\Rightarrow$ $41d = 164 \Rightarrow d = 4$

When $d = 4$, eq. (i) becomes

$a = \frac{5}{4} \times 4 \Rightarrow a = 5$

Now, sum of first 15 terms, $S_{15} = \frac{15}{2}(2a + 14d)$

$= \frac{15}{2}(2 \times 5 + 14 \times 4)$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$= \frac{15}{2} \times 66 = 15 \times 33 = 495$

Question 33.

The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms.

Solution:

Let 1^{st} term of the AP = a and Common difference = d

A.T.Q., $a_{13} = 4 \times a_3$ [Given]

$a + 12d = 4(a + 2d)$

$a + 12d = 4a + 8d \Rightarrow 3a = 4d$

$a = \frac{4}{3}d$ ...(i)

Also $a_5 = 16 \Rightarrow a + 4d = 16$

$\Rightarrow$ $\frac{4}{3}d + 4d = 16$ [Using (i)]

$\Rightarrow$ $16d = 48 \Rightarrow d = 3$

When $d = 3$, (i) becomes $a = \frac{4}{3} \times 3 = 4$

$\Rightarrow$ $a = 4$

Now, sum of first 10 terms, $S_{10} = \frac{10}{2}(2a + 9d)$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$= 5(2 \times 4 + 9 \times 3) = 5 \times 35 = 175.$

Question 34.

In an A.P., if the 12th term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Solution:

Let first term of the AP = a and common difference = d.

Let 2^{nd}, 3^{rd}, 4^{th} term be $a + d$, $a + 2d$, $a + 3d$ respectively.

Now, given $a_{12} = -13$

$\Rightarrow$ $a + 11d = -13 \Rightarrow a = -13 - 11d$ ...(i)

Also, $a + a + d + a + 2d + a + 3d = 24$ [$\because$ Sum of first four terms = 24]

$\Rightarrow$ $4a + 6d = 24$

$\Rightarrow$ $4(-13 - 11d) + 6d = 24$

$\Rightarrow$ $-52 - 44d + 6d = 24 \Rightarrow -38d = 76$

$d = -2$

$\therefore$ $a = -13 + 22 = 9$

$\therefore$ Sum of first ten terms, $S_{10} = \frac{10}{2}(2a + 9d)$

$= 5(2 \times 9 + 9 \times -2) = 0$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

Question 35.

Ramkali required ? 2500 after 12 weeks to send her daughter to school. She saved t 100 in the first week and increased her weekly saving by ? 20 every week. Find whether she will be able to send her daughter to school after 12 weeks or not. What value is generated in the above situation?

Solution:

Here, first term $a = 100$ and common difference $d = 20$

Now, savings after 12 weeks $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{12} = \frac{12}{2}[2 \times 100 + 20(12-1)] = 6(200 + 220) = 6 \times 420$

$= 2520$

So, Ramkali saved ₹ 2520 in 12 weeks and she required ₹ 2500 only.

$\therefore$ She will be able to send her daughter to school.

Ramkali is very much concerned about her daughter's education. She is awared and dedicated about giving education.

Question 36.

Find the 60th term of the AP 8,10,12,…, if it has a total of 60 terms and hence find the sum of its last 10 terms.

Solution:

AP is 8, 10, 12, ...

First term $a = 8$, common difference $d = 2$

We know, n^{th} term of A.P. $= a + (n-1)d$

$a_{60} = a + 59d = 8 + 59 \times 2 = 8 + 118 = 126$

As, $S_n = \frac{n}{2}[2a + (n-1)d]$

So, $S_{60} = \frac{60}{2}(a + a_{60}) = 30(8 + 126) = 30 \times 134 = 4020$

$S_{50} = \frac{50}{2}(2a + 49d) = 25(16 + 49 \times 2) = 25(114) = 2850$

$\therefore$ Sum of last 10 terms $= S_{60} - S_{50} = 4020 - 2850 = 1170$

Question 37.

An arithmetic progression 5,12,19,… has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Solution:

Given, A.P. is 5, 12, 19, ...

Now, first term $a = 5, d = 7, n = 50$

Now, $a_{50} = a + 49d = 5 + 49 \times 7 = 348$

$\therefore$ $S_{50} = \frac{50}{2}(a + a_{50}) = 25(5 + 348) = 8825$

$\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\therefore$ Sum of last 15 terms $= S_{50} - S_{35}$, where

$S_{35} = \frac{35}{2}(2 \times 5 + 34 \times 7) = 4340$

$\therefore$ Sum of last fifteen terms $= 8825 - 4340 = 4485.$

Question 38.

Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately.

Solution:

$$\text{Now,} \quad a_n = 999 \Rightarrow a + (n-1)d = 999$$
$$103 + (n-1)4 = 999 \Rightarrow (n-1)4 = 896$$
$$\Rightarrow \quad n - 1 = 224 \Rightarrow n = 225$$

Since, number of terms is odd, so there will be only one middle term.

$$\text{Middle term} = \frac{225+1}{2} = 113 = \left(\frac{n+1}{2}\right)^{th}$$
$$\therefore \quad a_{113} = a + 112d = 103 + 112 \times 4 = 103 + 448 = 551$$

There are 112 numbers before 113th term, where

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ Sum of all terms before middle term

$$S_{112} = \frac{112}{2}[2 \times 103 + 111 \times 4] = 56[206 + 444] = 36400$$

$\therefore$ Sum of all terms $= S_{225} = 123975$

$\therefore$ Sum of terms after middle term $= S_{225} - (S_{112} + 551) = 87024$

Question 39.

Find the middle term of the sequence formed by all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

Solution:

List of number between 9 and 95 leaving remainder 1, when divided by 3 are 10, 13, 16, ... 94

These numbers are in AP with

$$a = 10, d = 3$$

$\because$ number of terms in AP $= n$,

$$a_n = 94 \Rightarrow a + (n-1)d = 94$$
$$10 + (n-1)3 = 94$$
$$(n-1)3 = 84 \Rightarrow n - 1 = 28$$
$$n = 29$$

Since number of terms is odd, it has only one middle term.

$$\therefore \text{ Now,} \quad \text{Middle term} = \frac{29+1}{2} = 15^{th} \text{ term} = \left(\frac{n+1}{2}\right)^{th}$$
$$a_{15} = a + 14d = 10 + 14 \times 3 = 52$$

Number of terms before 15th term = 14

$$\therefore \text{ Sum of first 14 terms,} \quad S_{14} = \frac{14}{2}(2 \times 10 + 13 \times 3) \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$
$$= \frac{14}{2}(20 + 39) = 7 \times 59 = 413$$
$$a_{29} = 94$$
$$\therefore \quad S_{29} = \frac{29}{2}[a + a_{29}] = 1508$$
$$\therefore \quad \text{Sum of last 14 terms} = S_{29} - [S_{14} + a_{15}] = 1043$$

Question 40.

Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately.

Solution:

List of three digit number that leaves a remainder of 5, when divided by 7 are 103, 110, 117, ... 999.

These numbers are in AP with

$a = 103, d = 7, a_n = 999$, where n = number of terms

$\Rightarrow \quad a + (n-1)d = 999$

$\Rightarrow \quad 103 + (n-1)7 = 999 \Rightarrow (n-1)7 = 896$

$\Rightarrow \quad n - 1 = 128 \Rightarrow n = 129$

Since number of terms is odd, so only one middle term

$\therefore \quad \text{Middle term} = \frac{129+1}{2} = 65\text{th} = \left(\frac{n+1}{2}\right)^{\text{th}}$

$\therefore \quad a_{65} = a + 64d = 103 + 64 \times 7 = 103 + 448 = 551$

Number of terms before 65th term = 64

$\therefore \quad S_{64} = \frac{64}{2}(2 \times 103 + 63 \times 7) \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$= 32(206 + 441) = 20704$

$a_{129} = 999 = a + 128d$

$\therefore \quad S_{129} = \frac{129}{2}[a + a_{129}] = 71079$

Now, sum of terms after middle term $= S_{129} - (S_{64} + 551) = 49824$

2014

Short Answer Type Questions I [2 Marks]

Question 41.

The first and the last terms of an AP are 8 and 65 respectively. If sum of all its terms is 730, find its common difference.

Solution:

Hence, first term, $a = 8$; n^{th} term, $a_n = 65$; $S_n = 730$.

Now, we know that $\quad S_n = \frac{n}{2}(a + a_n)$

$730 = \frac{n}{2}(8 + 65)$

$\Rightarrow \quad \frac{73n}{2} = 730 \Rightarrow n = 20$

$\therefore$ Given, $\quad a_{20} = 65$, where $a_n = a + (n-1)d$

$\Rightarrow \quad a + 19d = 65 \Rightarrow 8 + 19d = 65$

$\Rightarrow \quad 19d = 57$

Hence, common differences, $d = 3$.

Question 42.

The first and the last terms of an AP are 7 and 49 respectively. If sum of all its terms is 420, find its common difference.

Solution:

First term, $a = 7$; n^{th} term, $a_n = 49$; $S_n = 420$.

Now, $\quad S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow \quad 420 = \frac{n}{2}(7 + 49)$

$\Rightarrow \quad 840 = 56n \Rightarrow n = 15$

$\therefore$ Given, $\quad a_{15} = 49$, where $a_n = a + (n-1)d$

$\Rightarrow \quad a + 14d = 49 \Rightarrow 7 + 14d = 49$

$\Rightarrow \quad 14d = 42 \Rightarrow d = 3$

Hence, common difference, $d = 3$.

Question 43.

The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is

400, find its common difference.

Solution:

Here, first term, $a = 5$; n^{th} term, $a_n = 45$; $S_n = 400$.

Now, $S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow 800 = 50n \Rightarrow n = 16$

$\therefore$ Given, $a_{16} = 45$, where $a_n = a + (n + 1)d$

$\Rightarrow a + 15d = 45 \Rightarrow 5 + 15d = 45$

$\Rightarrow 15d = 40$

$\Rightarrow d = \frac{8}{3}$

Hence, common difference, $d = \frac{8}{3}$.

Question 44.

Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution:

Numbers between 101 and 999 which are divisible by both 2 and 5 (*i.e.* by 10) are 110, 120, 130, ... 990.

An A.P. is formed with $a = 110$, $d = 10$ and $a_n = 990$

Now, we know that $a_n = a + (n - 1)d$

$\Rightarrow 990 = 110 + (n - 1)10$

$\Rightarrow 880 = (n - 1)10$

$\Rightarrow 88 = n - 1$

$\Rightarrow n = 89$

$\therefore$ Natural numbers which are divisible by 2 and 5 both are 89.

Question 45.

The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P.

Solution:

Given, Sum of first 'n' terms of AP $S_n = 3n^2 + 6n$

Replacing 'n' by $(n - 1)$

So,
$$\begin{aligned} S_{n-1} &= 3(n-1) + 6(n-1) \\ &= 3(n^2 - 2n + 1) + 6n - 6 \\ &= 3n^2 - 6n + 3 + 6n - 6 \\ &= 3n^2 - 3 \end{aligned}$$

Let n^{th} terms of AP be a_n.

Now,
$$\begin{aligned} a_n &= n^{th} \text{ term} = S_n - S_{n-1} \\ &= 3n^2 + 6n - 3n^2 + 3 \\ &= 6n + 3 \end{aligned}$$

Question 46.

The sum of the first n terms of an AP is $5n – n^2$. Find the nth term of this AP.

Solution:

Given, sum of first 'n' terms of AP is

$$S_n = 5n - n^2$$

Replacing 'n' by $(n - 1)$

So,
$$\begin{aligned} S_{n-1} &= 5(n-1) - (n-1)^2 = 5n - 5 - (n^2 - 2n + 1) \\ &= 5n - 5 - n^2 + 2n - 1 = 7n - n^2 - 6 \end{aligned}$$

Now,
$$\begin{aligned} a_n &= S_n - S_{n-1} = n^{th} \text{ term} = 5n - n^2 - 7n + n^2 + 6 \\ a_n &= 6 - 2n \end{aligned}$$

Question 47.

The sum of the first n terms of an AP is $4n^2 + 2n$. Find the nth term of this AP.

Solution:

Given; $S_n = 4n^2 + 2n$

So, $S_{n-1} = 4(n-1)^2 + 2(n-1) = 4(n^2 - 2n + 1) + 2n - 2$

$= 4n^2 - 8n + 4 + 2n - 2 = 4n^2 - 6n + 2$

$\therefore$ $a_n = S_n - S_{n-1} = n^{th}$ term $= (4n^2 + 2n) - (4n^2 - 6n + 2)$

$= 4n^2 + 2n - 4n^2 + 6n - 2 = 8n - 2.$

Short Answer Type Questions II [3 Marks]

Question 48.

If the seventh term of an AP is and its ninth term is , find its 63rd term.

Solution:

Let 'a' be the first term and 'd' be the common difference of an AP.

Here, $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ ...(i) $[\because a_n = a + (n-1)d]$

and $a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7}$ ...(ii)

Subtracting eq. (i) from eq. (ii), we get

Now, $2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$

Putting $d = \frac{1}{63}$ in eqn (i), we get

$$a + 6 \times \frac{1}{63} = \frac{1}{9} \Rightarrow a = \frac{1}{9} - \frac{6}{63} = \frac{7-6}{63} = \frac{1}{63}$$

$\Rightarrow$ $a = \frac{1}{63}$

Now, $a_{63} = a + 62d = \frac{1}{63} + \frac{62}{63} = \frac{63}{63} = 1$

Hence, 63rd term is 1.

Question 49.

The sum of the 2nd and the 7th terms of an AP is 30. If its 15th term is I less than twice its 8th term, find the AP.

Solution:

Let a be the first term and d be the common difference of a given A.P.

Given, $a_2 + a_7 = 30$

$\Rightarrow$ $a + d + a + 6d = 30 \Rightarrow 2a + 7d = 30$ ...(i) $[\because a_n = a + (n-1)d]$

Also, given $a_{15} = 2a_8 - 1$

$\Rightarrow$ $a + 14d = 2(a + 7d) - 1$

$\Rightarrow$ $a + 14d = 2a + 14d - 1 \Rightarrow a = 1$

Putting the value of a in (i), we get

$$2 + 7d = 30 \Rightarrow 7d = 28 \Rightarrow d = 4$$

$\therefore$ $a = 1, d = 4$

Hence, A.P. is 1, 5, 9, 13, 17, ...

Question 50.

The sum of the first seven terms of an AP is 182. If its 4tji and the 17th terms are in the ratio 1: 5, find the AP.

Solution:

Let a be the first term and d be the common difference of a given A.P.

According to question, $S_7 = 182$, where $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad \frac{7}{2}[2a + (7-1)d] = 182$

$\Rightarrow \quad a + 3d = 26 \quad ...(i)$

Also, given $\frac{a_4}{a_{17}} = \frac{1}{5}$

$\Rightarrow \quad \frac{a+3d}{a+16d} = \frac{1}{5}$, where $a_n = a + (n-1)d$

$\Rightarrow \quad 5a + 15d = a + 16d$

$\Rightarrow \quad 4a = d \quad ...(ii)$

From (i) and (ii), we get

$a + 12a = 26$

$\Rightarrow \quad 13a = 26$

$\Rightarrow \quad a = 2$

From (ii), we get $d = 8$

$\therefore \quad a = 2$ and $d = 8$

$\therefore$ A.P. is 2, 10, 18, ...

Question 51.

The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP.

Solution:

Let a be the first term and d be the common difference of a given A.P.

Given, $a_5 + a_9 = 30$, where $a_n = a + (n-1)d$

$\Rightarrow \quad a + 4d + a + 8d = 30$

$\Rightarrow \quad 2a + 12d = 30 \Rightarrow a + 6d = 15 \quad ...(i)$

Also, given $a_{25} = 3a_8$

$\Rightarrow \quad a + 24d = 3(a + 7d)$

$\Rightarrow \quad a + 24d = 3a + 21d$

$\Rightarrow \quad 3d = 2a \quad ...(ii)$

From (i) and (ii), $a + 4a = 15 \Rightarrow 5a = 15 \Rightarrow a = 3$

$\therefore$ From (ii), we get $3d = 2 \times 3 \Rightarrow d = 2$

$\therefore \quad a = 3, d = 2$

Hence, A.P. is 3, 5, 7, 9, ...

Question 52.

The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161.Find the 28th term of this AP.

Solution:

Given: $S_7 = 63$

where $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad a_1 + a_2 + ... + a_7 = 63$

$\Rightarrow \quad \frac{7}{2}(2a + 6d) = 63 \Rightarrow a + 3d = 9 \quad ...(i)$

Now, given $a_8 + a_9 + ... + a_{14} = 161$ [$\because$ Sum of next 7 terms is 161]

$\Rightarrow \quad S_{14} - S_7 = 161 \Rightarrow S_{14} = 161 + S_7$

$\Rightarrow \quad \frac{14}{2}(2a + 13d) = 161 + 63 \Rightarrow 7(2a + 13d) = 224$

$\Rightarrow \quad 2a + 13d = 32 \quad ...(ii)$

On solving the equations (i) and (ii), we get

$a = 3$ and $d = 2$

Now, $a_{28} = a + 27d = 3 + 27 \times 2 = 57$ [$\because a_n = a + (n-1)d$]

Long Answer Type Questions [4 Marks]

Question 53.

In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

Solution:

Let first term be 'a' and common difference be d.

Given, $n = 50$

ATQ, $a_1 + a_2 + \ldots\ldots + a_{10} = 210 = S_{10}$

$\Rightarrow \frac{10}{2}(a_1 + a_{10}) = 210$ $\quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\therefore 5(a + a + 9d) = 210$

$\Rightarrow 2a + 9d = 42$ ...(i)

and $a_{36} + a_{37} + \ldots\ldots + a_{50} = 2565$ [$\because$ Sum of last 15 terms = 2565]

$\Rightarrow \frac{15}{2}(a_{36} + a_{50}) = 2565$

$\Rightarrow a + 35d + a + 49d = \frac{2565 \times 2}{15}$

$\Rightarrow 2a + 84d = 171 \times 2$

$\Rightarrow a + 42d = 171$ ...(ii)

Question 54.

In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?

Solution:

According to question, each section of:

Class I will plant 2 trees, class II will plant 4 trees, class III will plant 6 trees and so on.. class 12 will plant 24 trees and each class has 2 sections.

$\therefore$ Number of trees planted = $4 + 8 + 12 + \cdots + 48$

This forms an A.P. with $a = 4$, $d = 4$ and $n = 12$

$\therefore$ Number of trees planted, $S_{12} = \frac{12}{2}(4 + 48) = 6 \times 52 = 312$ $\quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

Students are concerned about safety and pollution free environment.

Question 55.

If Sn denotes the sum of the first n terms of an A.P., prove that S30 = 3(S20-S10)

Solution:

Let sum of first 'n' terms of A.P., $S_n = \frac{n}{2}[2a + (n-1)d]$

where $a \rightarrow$ first term of A.P.

$d \rightarrow$ common difference of A.P.

$$\begin{aligned} \text{RHS} = 3(S_{20} - S_{10}) &= 3\left[\frac{20}{2}\{2a + 19d\} - \frac{10}{2}\{2a + 9d\}\right] \\ &= 3[20a + 190d - 10a - 45d] \\ &= 3[10a + 145d] = 15[2a + 29d] \\ &= \frac{30}{2}[2a + (30-1)d] = S_{30} = \text{LHS} \end{aligned}$$

2013

Short Answer Type Questions

Question 56.

How many three digit natural numbers are divisible by 7?

Solution:

Three digit natural numbers which are divisible by 7 are 105, 112, 119, ... 994.

$\because$ above is AP. Here $a = 105;\ d = 7$

Let $a_n = 994$

$\Rightarrow a + (n-1)d = 994$

$\Rightarrow 105 + 7(n-1) = 994 \Rightarrow 7n + 98 = 994$

$\Rightarrow 7n = 896 \Rightarrow n = 128$

Hence, there are 128 natural numbers of 3-digit which are divisible by 7.

Question 57.

Find the number of all three-digit natural numbers which are divisible by 9.

Solution:

3-digit numbers which are divisible by 9 are 108, 117, 126 .... 999.

Here, $a = 108, a_n = 999, d = 9.$

$\therefore$ Then, $a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1)9$

$999 - 108 = (n-1)9 \Rightarrow 891 + 9 = 9n$

$\Rightarrow \frac{900}{9} = n \Rightarrow n = 100$

$\therefore$ There are 100 three-digit natural numbers which are divisible by 9.

Question 58.

Find the number of three-digit natural numbers which are divisible by 11

Solution:

Three-digit natural numbers divisible by 11 are 110, 121, 132, ..., 990

These form an AP with $a = 110$ and $d = 121 - 110 = 11$.

Last term $= 990 = a_n$. Then

$\Rightarrow a_n = 990$

$\Rightarrow a + (n-1)d = 990$

$\Rightarrow 110 + (n-1)11 = 990 \Rightarrow 11(n-1) = 880$

$\Rightarrow n - 1 = 80 \Rightarrow n = 81$

$\therefore$ There are 81 three-digit natural numbers which are divisible by 11.

Short Answer Type Questions II [3 Marks]

Question 59.

Find the number of terms of the AP: 18,15. 1/2, 13……..(-49. 1/2),and find the sum of all its terms.

Solution:

Given AP is: 18, $15\frac{1}{2}$, 13, ... $\left(-49\frac{1}{2}\right)$

Here, $a = 18;\ d = \frac{31}{2} - 18 = \frac{-5}{2}$

Let n^{th} term, $a_n = -\left(49\frac{1}{2}\right)$

$\Rightarrow a + (n-1)d = -\frac{99}{2}$

$\Rightarrow 18 - \frac{5}{2}(n-1) = -\frac{99}{2} \Rightarrow 18 - \frac{5n}{2} + \frac{5}{2} = -\frac{99}{2}$

$\Rightarrow \frac{5n}{2} = 18 + \frac{5}{2} + \frac{99}{2}$

$\Rightarrow 5n = 36 + 5 + 99 \Rightarrow 5n = 140$

$\Rightarrow n = 28.$

Now, sum of all terms, $S_{28} = \frac{28}{2}\left[2 \times 18 + 27 \times \left(-\frac{5}{2}\right)\right] = 14\left[36 - \frac{135}{2}\right],$

where $S_n = \frac{n}{2}[2a + (n-1)d] = 14\left(\frac{72 - 135}{2}\right) = 7 \times (-63) = -441$

Question 60.

The nth term of an AP is given by (-4n + 15). Find the sum of first 20 terms of this A.Progressions

Solution:

Here, given $a_n = -4n + 15 = n^{th}$ term

So, $a_1 = -4 \times 1 + 15 = 11$

$a_2 = -4 \times 2 + 15 = 7$

$d = a_2 - a_1 = 7 - 11 = -4$

$\because$ Sum of 'n' terms $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{20} = \frac{20}{2}[2 \times 11 - 4(20-1)] = 10(22 - 4 \times 19)$

$= 10(22 - 76) = 10 \times (-54) = -540.$

Question 61.

The sum of first n-terms of an AP is $3n^2 + 4n$. Find the 25th term of this AP.

Solution:

Given: Sum of first n terms, $S_n = 3n^2 + 4n$

so, $S_1 = 3(1^2) + 4(1) = 3 + 4 = 7$

$\therefore$ First term, $a_1 = 7$ $[\because a_1 = S_1]$

$S_2 = 3(2)^2 + 4(2) = 20$

Now, $a_2 = S_2 - S_1 = 20 - 7 = 13$

$\therefore$ Common difference $d = a_2 - a_1 = 13 - 7 = 6$

Now, 25^{th} term, $a_{25} = a + 24d = 7 + 24 \times 6 = 7 + 144 = 151.$

Hence, $a_{25} = 151.$

Question 62.

The 8th term of an AP is equal to three times its 3rd term. If its 6th term is 22, find the AP.

Solution:

Consider 1st term $= a$

Common difference $= d$

Given that $a_8 = 3a_3$, where $a_n = a + (n-1)d$

$a + 7d = 3(a + 2d)$

$a + 7d = 3a + 6d$

$7d - 6d = 3a - a$

$d = 2a$ ...(i)

Now, $a_6 = a + 5d$

$22 = a + 5d$ $[\because$ Given that $a_6 = 22]$

$22 = a + 5(2a)$ [Using (i)]

$22 = 11a$

$a = 2$

$\therefore$ $d = 2a = 2(2) = 4$

Hence, required AP 2, 6, 10, 14 .....

Question 63.

The 9th term of an AP is equal to 6 times its 2nd term. If its 5th term is 22, find the AP.

Solution:

Let Ist term be a

Common difference $= d$

$\therefore$ Given that, $a_9 = 6a_2$, where $a_n = a + (n-1)d$

$$a + 8d = 6(a + d)$$
$$a + 8d = 6a + 6d$$
$$2d = 6a - a$$
$$2d = 5a$$
$$d = \frac{5}{2}a$$

Now, $a_5 = a + 4d$

$$22 = a + 4\left(\frac{5}{2}a\right) \quad \text{[Given that } a_5 = 22]$$
$$22 = a + 10a$$
$$11a = 22 \Rightarrow a = 2$$

$\therefore$ $d = 5$

Required AP is 2, 7, 12, 17, 22 ...

Question 64.

The 19th term of an AP is equal to three times its 6th term. If its 9th term is 19, find the AP.

Solution:

Let Ist term of the AP = a and common difference = d.

A.T.Q., $a_{19} = 3 \times a_6$, where $a_n = a + (n-1)d$

$\Rightarrow$ $a + 18d = 3(a + 5d) \Rightarrow a = \frac{3}{2}d$ ...(i)

Also, given that $a_9 = 19$

$\Rightarrow$ $a + 8d = 19$

$\Rightarrow$ $\frac{3}{2}d + 8d = 19$ [Using eq. (i)]

$\Rightarrow$ $19d = 38 \Rightarrow d = 2$

Putting $d = 2$, equation (i), we get

$$a = \frac{3}{2} \times 2 = 3$$

$\therefore$ Required AP is 3, 5, 7, 9, ...

Question 65.

The 8th term of an AP is 31. If its 15th term exceeds its 11th term by 16, find the AP.

Solution:

Consider Ist term = a

Common difference = d

Now, given that $a_8 = 31$, where $a_n = a + (n-1)d$

$\Rightarrow$ $a + 7d = 31$ ...(i)

Also, given $a_{15} - a_{11} = 16$

$\Rightarrow$ $(a + 14d) - (a + 10d) = 16 \Rightarrow 14d - 10d = 16$

$\Rightarrow$ $4d = 16 \Rightarrow d = 4$

$\therefore$ From (i), $a + 7(4) = 31$

$\Rightarrow$ $a + 28 = 31 \Rightarrow a = 31 - 28 = 3$

$\Rightarrow$ $a = 3$

$\therefore$ Required AP is 3, 7, 11, 15, ...

Question 66.

The 18th term of an AP is 30 more than its 8th term. If the 15th term of the AP is 48, find the AP.

Solution:

Consider Ist term = a

Common difference = d

As per condition, $a_{18} = a_8 + 30$, where $a_n = a + (n-1)d$

$$a + 17d = a + 7d + 30$$
$$17d - 7d = 30$$
$$10d = 30 \Rightarrow d = 3$$

Also, $a + 14d = a_{15}$

$\Rightarrow$ $a_{15} = 48$ (given)

$\therefore$ $a + 14d = 48$

$$a + 14(3) = 48$$
$$a + 42 = 48$$
$$a = 6$$

$\therefore$ Required AP is 6, 9, 12, 15 ...

Question 67.

The 5th term of an AP exceeds its 12th term by 14. If its 7th term is 4, find the AP.

Solution:

Let 1st term = a

Common difference = d

$\therefore$ As per condition, $a_5 = 14 + a_{12}$

$\Rightarrow$ $a + 4d = 14 + a + 11d$ $\quad [\because a_n = a + (n-1)d]$

$$4d - 11d = 14$$
$$-7d = 14 \Rightarrow d = -2$$

Also, given that $a_7 = 4$

$\Rightarrow$ $a + 6d = 4$

$$a + 6(-2) = 4$$
$$a - 12 = 4$$
$$a = 4 + 12 = 16$$

$\therefore$ Required AP is 16, 14, 12, 10 ...

Long Answer Type Questions [4 Marks]

Question 68.

Find the number of terms of the AP – 12, -9,-6, … 21. If 1 is added to each term of this AP, then find the sum of all terms of the AP thus obtained. [Delhi]

Solution:

Given AP is –12, –9, –6, ... 21 when 1 is added to each term of above AP, then new AP is – 11, – 8, –5, ... 22.

Here, $a = -11;\ d = 3$ and let $a_n = 22 = n^{th}$ term

$\Rightarrow$ $a + (n-1)d = 22$

$\Rightarrow$ $-11 + 3(n-1) = 22 \Rightarrow 3n - 3 = 33$

$\Rightarrow$ $3n = 36 \Rightarrow n = 12$

$\because$ Sum of all terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Now, $S_{12} = \frac{12}{2}[2 \times (-11) + 3 \times 11]$

$$= 6[-22 + 33] = 6 \times 11 = 66$$

Question 69.

The 24th term of an AP is twice its tenth term. Show that its 72nd term is 4times its 15th term.

Solution:

Given $a_{24} = 2a_{10}$

$\Rightarrow a + 23d = 2(a + 9d)$ [$\because a_n = a + (n-1)d$]

$\Rightarrow a + 23d = 2a + 18d \Rightarrow 5d = a$

Now, consider $a_{15} = a + 14d = 5d + 14d$ [using $a = 5d$]

$\Rightarrow a_{15} = 19d$ ...(i)

Now, consider $a_{72} = a + 71d = 5d + 71d = 76d = 4(19d)$

$\Rightarrow a_{72} = 4a_{15}$ [using (i)]

Question 70.

If the sum of first 7 terms of an AP is 49 and that of first 17 terms 289. find the sum of its first n terms.

Solution:

Let 'a' be the first term and 'd' be the common difference of an AP.

Given $S_7 = 49 =$ Sum of first 7 terms

$\Rightarrow \frac{7}{2}(2a + 6d) = 49$

$\Rightarrow 2a + 6d = 14 \Rightarrow a + 3d = 7$ ...(i)

Also, given that $S_{17} = 289$

$\Rightarrow \frac{17}{2}(2a + 16d) = 289$

$\Rightarrow 2a + 16d = 34 \Rightarrow a + 8d = 17$ ...(ii)

On solving the equations (i) and (ii) we get

$a = 1$ and $d = 2$

Now, sum of first 'n' terms, $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 1 + 2(n-1)]$

$= \frac{n}{2}(2 + 2n - 2) = \frac{n}{2} \times 2n = n^2$

Question 71.

The sum of first m terms of an AP is $4m^2 – m$. If its n. Also, find the 21st term of this AP.

Solution:

Given that $S_m = 4m^2 - m =$ Sum of first 'm' terms

n^{th} terms, $T_n = S_n - S_{n-1} = 4n^2 - n - [4(n-1)^2 - (n-1)]$

$= 4n^2 - n - [4n^2 + 4 - 8n - n + 1]$

$= 4n^2 - n - 4n^2 + 9n - 5 = 8n - 5$

But given $T_n = 107$

$\therefore 107 = 8n - 5 \Rightarrow 8n = 112$

$\Rightarrow n = \frac{112}{8} = 14$

$\therefore$ 21st term, $T_{21} = 8(21) - 5 = 168 - 5 = 163$

Question 72.

The sum of first q terms of an AP is $63q – 3q^2$. If its pth term is -60, find the value of p. Also find the 11th term of this AP.

Solution:

Given that $S_q = 63q - 3q^2 =$ Sum of first 'q' terms ...(i)

Now, p^{th} term, $T_p = S_p - S_{p-1} = [63p - 3p^2] - [63(p-1) - 3(p-1)^2]$

$= 63p - 3p^2 - (63p - 63 - 3p^2 - 3 + 6p)$

$= 63p - 3p^2 - 63p + 3p^2 - 6p + 66 = -6p + 66$

Also, given that $T_p = -60$

$\Rightarrow -6p + 66 = -60$

$\Rightarrow -6p = -60 - 66$

$\Rightarrow -6p = -126 \Rightarrow p = 21$

$\therefore$ 11^{th} term, $T_{11} = -6(11) + 66 = -66 + 66 = 0$

Question 73.

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. Find the total number of trees planted by the students of the school.
Pollution control is necessary for everybody's health. Suggest one more role of students in it.

Solution:

Number of trees planted by class I $= 3 \times 1 = 3$
Number of trees planted by class II $= 3 \times 2 = 6$
Number of trees planted by class III $= 3 \times 3 = 9$
Number of trees planted by class XII $= 3 \times 12 = 36$
Total number of trees planted by students

$$= 3 + 6 + 9 + \cdots + 36 \qquad [12 \text{ terms}]$$

$$= \frac{12}{2}(3 + 36) = 234 \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

Role of students for everybody's health. To provide safety and pollution-free environment.

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Short Answer Type Questions I [2 Marks]

Question 74.

Find the sum of all three digit natural numbers, which are multiples of 11.

Solution:

3 digit natural numbers which are multiples of 11 are 110, 121, 132, ..., 990

$$a = 110, a_n = l = 990, d = 11$$

n^{th} term, $a_n = a + (n-1)d$

$\Rightarrow 990 = 110 + (n-1)11 \Rightarrow 880 = (n-1)11$

$\Rightarrow 80 = n - 1 \Rightarrow n = 81$

$\therefore$ Sum of 'n' terms, $S_n = \frac{n}{2}[a + l]$

$$= \frac{81}{2}[110 + 990] = \frac{81}{2} \times 1100 = 44550$$

$\therefore$ Sum of all three-digit natural numbers, which are multiples of 11 is 44550.

Question 75.

Find the sum of all three digit natural numbers, which are multiples of 9.

Solution:

3-digit natural numbers which are multiples of 9 are 108, 117, ..., 999
It form an AP with $a = 108, d = 9, a_n = 999$

$\therefore$ n^{th} term, $a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1) \times 9 \Rightarrow 999 - 108 = (n-1) \times 9$

$\Rightarrow (n-1) = \frac{891}{9} = 99 \Rightarrow n = 99 + 1 = 100$

$$S_{100} = \frac{100}{2}(108 + 999) = 55350$$

$\therefore$ Sum of all 3-digit natural numbers, multiples of 9 is 55350.

Question 76.

Find the sum of all three digits natural numbers, which are multiples of 7.

Solution:

3-digit natural numbers, which are multiples of 7 are 105, 112, 119, ..., 994

Here $a = 105; d = 7; a_n = 994 = n^{th}$ term

Now, $a_n = 994$

$\Rightarrow a + (n-1)d = 994 \Rightarrow 105 + 7(n-1) = 994$

$\Rightarrow 7(n-1) = 889 \Rightarrow n - 1 = 127$

$\Rightarrow n = 128$

Now, sum of 128 terms, $S_{128} = \frac{128}{2}[2 \times 105 + 127 \times 7]$, where $S_n = \frac{n}{2}[2a + (n-1)d]$

$= 64 \times 1099 = 70336$

$\therefore$ Sum of all 3-digit natural numbers, which are multiples of 7 is 70336.

Question 77.

How many three digit numbers are divisible by 11?

Solution:

Three digit numbers which are divisible by 11 are 110, 121, 132, ..., 990

Here, $a = 110, d = 11, a_n = 990 = n^{th}$ term

Now, $a_n = 990$

$\Rightarrow a + (n-1)d = 990 \Rightarrow 110 + 11(n-1) = 990$

$\Rightarrow 11(n-1) = 880 \Rightarrow n - 1 = 80$

$\Rightarrow n = 81$

Hence, there are 81 three digit numbers which are divisible by 11.

Question 78.

How many three-digit numbers are divisible by 12?

Solution:

The three digit numbers divisible by 12 are 108, 120, 132, ..., 996

Here, $a = 108, d = 12, a_n = 996 = n^{th}$ term

Now, $a_n = a + (n-1)d$

$\Rightarrow 996 = 108 + (n-1)12$

$\Rightarrow 996 - 108 = (n-1)12 \Rightarrow 888 = (n-1)12$

$\Rightarrow 74 = n - 1 \Rightarrow n = 75$

$\therefore$ There are 75 three-digit numbers divisible by 12.

Question 79.

In an AP, the first term is 12 and the common difference is 6. If the last term of the A.P. is 252, find its middle term.

Solution:

Here $a = 12, d = 6$.

Let number of terms be n

So, $a_n = 252 =$ last term

$\Rightarrow a + (n-1)d = 252 \Rightarrow 12 + (n-1)6 = 252$

$\Rightarrow (n-1)6 = 240 \Rightarrow n - 1 = 40$

$\Rightarrow n = 41$

$\therefore$ Since number of terms is odd, so only one middle term.

Now, middle term $= \left(\frac{41+1}{2}\right) = 21\text{st term} = \left(\frac{n+1}{2}\right)^{th}$ term

$\therefore$ 21^{st} term, $a_{21} = a + 20d$

$= 12 + 20 \times 6 = 132 =$ middle term value.

Question 80.

In an A.P., the first term is 8 and the common difference is 7. If the last term of the A.P. is 218, find its middle term.

Solution:

Here, $a = 8, d = 7, a_n = 218$ = last term, Then,

$$a_n = a + (n-1)d$$

$\Rightarrow$ $218 = 8 + (n-1)7 \Rightarrow 210 = 7(n-1)$

$\Rightarrow$ $30 = n - 1 \Rightarrow n = 31$

$\therefore$ Since number of terms is odd, so only one middle term.

$\therefore$ middle term $= \left(\frac{31+1}{2}\right)^{th} = 16^{th}$ term

and 16^{th} term $a_{16} = a + (16-1)d$

$= 8 + 15 \times 7 = 8 + 105 = 113$

Question 81.

In an A.P., the first term is 5 and the common diference is 2. If the last term of the A.P. is 53, find its middle term.

Solution:

Here, first term $a = 5$; common difference, $d = 2$

last term, $a_n = 53$

$\Rightarrow$ $a + (n-1)d = 53$

$\Rightarrow$ $5 + (n-1) \times 2 = 53 \Rightarrow 2n - 2 = 53 - 5$

$\Rightarrow$ $2n - 2 = 48 \Rightarrow 2n = 48 + 2 = 50$

$\Rightarrow$ $n = \frac{50}{2} = 25$

There are 25 terms in an A.P. Since number of terms is odd, so only one middle term.

$\therefore$ middle term $= \left(\frac{25+1}{2}\right)^{th} = 13^{th}$

So, middle term $= T_{13}$

$= a + 12d = 5 + 12 \times 2 = 29$

Short Answer Type Questions II [3 Marks]

Question 82.

The 15th term of an A.P. is 3 more than twice its 7th term. If the 10th term of the A.P. is 41, then find its nth term.

Solution:

Let the A.P. has first term $= a$

Common difference $= d$

According to question, $a_{10} = 41$

$a + (10-1)d = 41$

$\Rightarrow$ $a + 9d = 41 \Rightarrow a = 41 - 9d$ ...(i)

$\Rightarrow$ Also given $a_{15} = 3 + 2a_7$

$\Rightarrow$ $a + 14d = 3 + 2(a + 6d) \Rightarrow a + 14d = 3 + 2a + 12d$

$\Rightarrow$ $14d - 12d = 2a - a + 3 \Rightarrow 2d = a + 3$

$\Rightarrow$ $2d = 41 - 9d + 3$ [using equation (i)]

$\Rightarrow$ $11d = 44$

$\Rightarrow$ $d = 4$

$\Rightarrow$ Then, we have $a = 41 - 9 \times 4 \Rightarrow a = 41 - 36 = 5$ [using equation (i)]

$\therefore$ nth term $= a_n = a + (n-1)d$

$= 5 + (n-1)4 = 5 + 4n - 4$

$\therefore$ n^{th} term $a_n = 4n + 1$

Question 83.

The 17th term of an A.P. is 5 more than twice is 8th term, if the 11th term of the A.P. is 43, then find its nth term.

Solution:

Given: $a_{11} = 43$, where $a_n = a + (n-1)d$

$\Rightarrow$ $43 = a + (11-1)d \Rightarrow 43 = a + 10d$ ...(*i*)

Also, $a_{17} = 2a_8 + 5$

$a + (17-1)d = 2[a + (8-1)d] + 5 \Rightarrow a + 16d = 2a + 14d + 5$

$2d - 5 = a$ ...(*ii*)

From (*i*) and (*ii*), we get

$43 = 2d - 5 + 10d$

$\Rightarrow$ $48 = 12d \Rightarrow d = 4$

Putting $d = 4$ in (*i*), we get

$43 = a + 10 \times 4$

$\Rightarrow$ $43 = a + 40 \Rightarrow a = 3$

$\therefore$ n^{th} term, $a_n = a + (n-1)d$

$\Rightarrow$ $a_n = 3 + (n-1)4 \Rightarrow a_n = 3 + 4n - 4$

$\therefore$ n^{th} term, $a_n = 4n - 1$

Question 84.

The 16 term of an A.P. is 1 more than twice its 8th term. If the 12th term of the A.P. is 47, then find its nth term.

Solution:

According to question, $a_{16} = 2a_8 + 1$, where $a_n = a + (n-1)d$

$\Rightarrow$ $a + 15d = 2(a + 7d) + 1 \Rightarrow a + 15d = 2a + 14d + 1$

$\Rightarrow$ $d = a + 1$ ...(*i*)

and given that $a_{12} = 47$

$\Rightarrow$ $a + 11d = 47 \Rightarrow a + 11(a+1) = 47$ [Using (*i*)]

$\Rightarrow$ $12a + 11 = 47 \Rightarrow 12a = 36$

$\Rightarrow$ $a = 3$

Putting $a = 3$ in eqn (*i*), we get $d = 4$

Now, n^{th} term, $a_n = a + (n-1)d$

$= 3 + 4(n-1) = 4n - 1$

Question 85.

Find the sum of all multiples of 7 lying between 500 and 900.

Solution:

First multiple of 7 which is more than 500 is 504

$\therefore$ Multiples of 7 between 500 and 900 are 504, 511, 518, ... 896, which are in A.P.

Here, $a = 504$ and $d = 7$

Now, $a_n = 896$ = last term

$\Rightarrow$ $a + (n-1)d = 896 \Rightarrow 504 + (n-1) \times 7 = 896$

$\Rightarrow$ $(n-1) \times 7 = 392 \Rightarrow n - 1 = 56 \Rightarrow n = 57$

$\therefore$ Sum of these multiples is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{57} = \frac{57}{2}(504 + 896) = 39900$$

Question 86.

Find the sum of all multiples of 8 lying between 201 and 950.

Solution:

The numbers which are multiples of 8 lying between 201 and 950 are:
208, 216, 224, ..., 944

Here $a = 208; d = 8$; last term $a_n = 944$

Now, $a_n = 944$

$\Rightarrow a + (n-1)d = 944 \Rightarrow 208 + 8(n-1) = 944$

$\Rightarrow 8(n-1) = 736 \Rightarrow n - 1 = 92$

$\Rightarrow n = 93$

Now, sum of these multiples, $S_{93} = \frac{93}{2}(208 + 944)$ $\left[\because S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= \frac{93}{2} \times 1152 = 93 \times 576 = 53568$

Question 87.

Find the sum of all multiples of 9 lying between 400 and 800.

Solution:

Multiples of 9 between 400 and 800 are: 405, 414, 423, ..., 792

Here, $a = 405; d = 9$; last term $a_n = 792$

Now, $a_n = 792$

$\Rightarrow a + (n-1)d = 792 \Rightarrow 405 - 9(n+1) = 792$

$\Rightarrow 9(n-1) = 387 \Rightarrow n - 1 = 43$

$\Rightarrow n = 44$

Now, sum of these multiples, $S_{44} = \frac{44}{2}(405 + 792)$ $\left[\because S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= 22 \times 1197 = 26334$

Question 88.

Find the sum of first 40 positive integers divisible by 6

Solution:

List of first 40 positive integers divisible by 6 are 6, 12, 18, 24, ...

Here, $a = 6; d = 6; n = 40$

$S_{40} = \frac{40}{2}(2 \times 6 + 39 \times 6)$ $\because S_n = \frac{n}{2}[2a + (n-1)d]$

Question 89.

If 4 times the fourth term of an A.P. is equal to 18 times its 18th term, then find its 22nd term.

Solution:

According to question, $4a_4 = 18a_{18}$

$\Rightarrow 4(a + 3d) = 18(a + 17d)$, where $a_n = a + (n-1)d$

$\Rightarrow 4a + 12d = 18a + 306d \Rightarrow 14a + 294d = 0$

$\Rightarrow 14(a + 21d) = 0 \Rightarrow a + 21d = 0$

$\Rightarrow a_{22} = 0$ $[\because a_{22} = a + 21d]$

Hence, 22nd term is zero.

Long Answer Type Questions [4 Marks]

Question 90.

The sum of the first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term.

Solution:

Let the A.P. be $a_1, a_2, ..., a_n$

Common difference $= d$

Here, $a = 15$

$S_{15} = 750$

$a_{20} = ?$

Now, $S_{15} = 750$

$\therefore \quad \frac{15}{2}[2 \times 15 + (15-1)d] = 750 \qquad \left\{S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$30 + 14d = \frac{750 \times 2}{15}$

$14d = 100 - 30 = 70$

$d = 5$

$\therefore$ 20th term, $a_{20} = a + 19d$

$= 15 + 19 \times 5 = 15 + 95 = 110$

Question 91.

Sum of the first 20 terms of an A.P. is – 240, and its first term is 7. Find its 24th term.

Solution:

Given that, $S_{20} = -240$ and first term, $a = 7$

$\Rightarrow \quad \frac{20}{2}(2a + 19d) = -240$

$\Rightarrow \quad 10(2 \times 7 + 19d) = -240 \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\Rightarrow \quad 14 + 19d = -24 \Rightarrow 19d = -38$

$\Rightarrow \quad d = -2$

Now, 24th term, $a_{24} = a + 23d$

$= 7 + 23 \times (-2) = 7 - 46 = -39$

Question 92.

Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Solution:

Here, $n = 14, S_{14} = 1505, a = 10, a_{25} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$

$1505 = 7[20 + 13d]$

$215 = 20 + 13d \Rightarrow 195 = 13d$

$\Rightarrow \quad d = 15$

Using $a_n = a + (n-1)d$

$\therefore$ 25th term, $a_{25} = 10 + (25-1)15$

$= 10 + 24 \times 15 = 10 + 360 = 370$

Question 93.

Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

Solution:

First term of the AP, $a = 5$ and common difference $= d$

According to question,

$(a_1 + a_2 + a_3 + a_4) = \frac{1}{2}(a_5 + a_6 + a_7 + a_8)$

$\Rightarrow \quad [a + (a+d) + (a+2d) + (a+3d)] = \frac{1}{2}[(a+4d) + (a+5d) + (a+6d) + (a+7d)]$

$\Rightarrow \quad (4a + 6d) = \frac{1}{2}(4a + 22d)$

$\Rightarrow \quad 2 \times (4 \times 5 + 6d) = (4 \times 5 + 22d) \qquad [\because a = 5]$

$\Rightarrow \quad 40 + 12d = 20 + 22d \Rightarrow 10d = 20$

$\Rightarrow \quad d = 2$

Question 94.

If the sum of the first 7 terms of an A.P. is 119 and that of the first 17 terms is 714, find the sum of its first n-terms. [All India]

Solution:

According to question, $S_7 = 119$, where $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{7}{2}(2a + 6d) = 119 \Rightarrow a + 3d = 17$ ...(*i*)

and also given $S_{17} = 714$

$\Rightarrow \frac{17}{2}(2a + 16d) = 714 \Rightarrow a + 8d = 42$ ...(*ii*)

On solving (*i*) and (*ii*), we get

$$a = 2 \text{ and } d = 5$$

Now, sum of '*n*' term

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 2 + 5(n-1)] = \frac{n(5n-1)}{2}$$

Question 95.

A sum of ? 1600 is to be used to give ten cash prizes to students of a school for their over all academic performance. If each prize is ? 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let the ten cash prize amount is

$a, a-20, a-40, a-60, a-80, a-100, a-120, a-140, a-160, a-180$

According to question,

$$a + (a-20) + (a-40) + ... + (a-180) = 1600$$

$$\Rightarrow (a + a + a + ... + a) - (20 + 40 + ... + 180) = 1600$$

$$\Rightarrow 10a - 20(1 + 2 + 3 + ... + 9) = 1600$$

$$\Rightarrow 10a - 20 \times \frac{9 \times 10}{2} = 1600 \quad \left[\because 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}\right]$$

$$\Rightarrow 10a - 900 = 1600$$

$$\Rightarrow 10a = 2500$$

$$\Rightarrow a = 250$$

$\therefore$ Cash prize amounts are as:

₹ 250, 230, 210, 190, 170, 150, 130, 110, 90, 70

Question 96.

The sum of 4th and 8th terms of an A.P. is 24 and the sum of its 6th and 10th terms is 44. Find the sum of first ten terms of the A.P.

Solution:

Given that $a_4 + a_8 = 24$, where $a_n = a + (n-1)d$

$\Rightarrow a + (4-1)d + a + (8-1)d = 24$

$\Rightarrow a + 3d + a + 7d = 24 \Rightarrow 2a + 10d = 24$ ...(i)

and also given $a_6 + a_{10} = 44$

$\Rightarrow a + (6-1)d + a + (10-1)d = 44$

$\Rightarrow a + 5d + a + 9d = 44 \Rightarrow 2a + 14d = 44$ ...(ii)

From (i) and (ii), we get

$$24 - 10d + 14d = 44$$

$$4d = 44 - 24 \Rightarrow 4d = 20$$

$\Rightarrow d = 5$

Putting $d = 5$ in (i), we get

$$2a + 10 \times 5 = 24$$

$\Rightarrow 2a + 50 = 24 \Rightarrow 2a = -26$

$\Rightarrow a = -13$

Here, $n = 10, a = -13, d = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ Sum of first 10^{th} terms, $S_{10} = \frac{10}{2}[2 \times -13 + (10-1)5]$

$$= 5[-26 + 45] = 19 \times 5 = 95$$

Question 97.

The sum of the first five terms of an A.P. is 25 and the sum of-its next five terms is – 75. Find the 10th term of the A.P.

Solution:

Given: $a_1 + a_2 + a_3 + a_4 + a_5 = 25$ [$\because$ Sum of first 5 terms = 25]

$\Rightarrow S_5 = 25 \Rightarrow \frac{5}{2}(2a + 4d) = 25 \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\Rightarrow 2a + 4d = 10$ ...(i)

Also, $a_6 + a_7 + a_8 + a_9 + a_{10} = -75$ [$\because$ Sum of next 5 term = –75]

$\Rightarrow S_{10} - S_5 = -75 \Rightarrow S_{10} = -75 + S_5$

$\Rightarrow S_{10} = -75 + 25 \Rightarrow S_{10} = -50$

$\Rightarrow \frac{10}{2}(2a + 9d) = -50 \Rightarrow 2a + 9d = -10$ ...(ii)

Subtract eqn (i) from eqn (ii), we get

$$5d = -20 \Rightarrow d = -4$$

putting $d = -4$ in eqn (i), we get

$$a = 13$$

Now, 10^{th} term, $a_{10} = a + 9d = 13 + 9(-4) = 13 - 36 = -23$

Thus, $a_{10} = -23$

Question 98.

The sum of the third and seventh terms of an A.P. is 40 and the sum of its sixth and 14th terms is 70. Find the sum of the first ten terms of the A.P.

Solution:

Given: $a_3 + a_7 = 40$ [$\because$ Sum of third and seventh term = 40]

$\Rightarrow a + 2d + a + 6d = 40$

$\Rightarrow 2a + 8d = 40 \Rightarrow a + 4d = 20$ ...(*i*)

and also given $a_6 + a_{14} = 70$ [$\because$ Sum of 6^{th} and 14^{th} term = 70]

$\Rightarrow a + 5d + a + 13d = 70$

$\Rightarrow 2a + 18d = 70 \Rightarrow a + 9d = 35$ ...(*ii*)

Subtract eqn (*i*) from eqn (*ii*), we get

$$\begin{aligned} a + 9d &= 35 \\ a + 4d &= 20 \\ - \quad - &\quad - \\ \hline 5d &= 15 \Rightarrow d = 3 \end{aligned}$$

Put $d = 3$ in eqn (*i*), we get

$a + 4 \times 3 = 20 \Rightarrow a = 20 - 12 \Rightarrow a = 8$

Now, sum of first ten terms, $S_{10} = \frac{10}{2}[2 \times 8 + 9 \times 3]$ $\because S_n = \frac{n}{2}[2a + (n-1)d]$

$= 5 \times [16 + 27] = 5 \times 43 = 215$

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Short Answer Type Questions I [2 Marks]

Question 99.

Is -150 a term of the AP 17,12, 7, 2,…?

Solution:

Given AP is 17, 12, 7, 2, ...

Here, $a = 17, d = 12 - 17 = -5$

Let $a_n = -150$

$a + (n-1)d = -150 \Rightarrow 17 + (n-1)(-5) = -150$

$\Rightarrow (n-1)(-5) = -150 - 17 \Rightarrow (n-1)(-5) = -167$

$\Rightarrow n - 1 = \frac{167}{5} \Rightarrow n = \frac{167}{5} + 1$

Question 100.

Find the number of two-digit numbers which are divisible by 6.

Solution:

Two digit numbers which are divisible by 6 are 12, 18, 24, ..., 96

Here $a = 12$ and $d = 18 - 12 = 6$

$\because$ last term, $a_n = 96 \Rightarrow 12 + (n-1)6 = 96$, where $a_n = a + (n-1)d$

$\Rightarrow (n-1)6 = 96 - 12 = 84$

$\Rightarrow n - 1 = \frac{84}{6} \Rightarrow n - 1 = 14$

$\Rightarrow n = 14 + 1 \Rightarrow n = 15$

$\therefore$ There are 15 two-digit numbers divisible by 6.

Question 101.

Which term of the A.P. 3,14,25,36,… will be 99 more than its 25th term

Solution:

Given A.P. is 3, 14, 25, 36, ...

Here $a = 3; d = 11$

Let a_n is the term which is 99 more than 25th term of above A.P.

A.T.Q. $a_n = a_{25} + 99$

$\Rightarrow a + (n-1)d = a + 24d + 99$

$\Rightarrow 11(n-1) = 24 \times 11 + 99$

$\Rightarrow 11(n-1) = 11(24 + 9)$

$\Rightarrow n - 1 = 33 \Rightarrow n = 34$

Hence, 34^{th} is the required term.

Question 102.

How many natural numbers are there between 200 and 500, which are divisible by 7?

Solution:

Natural numbers between 200 and 500 which are divisible by 7 are as

203, 210, 217, ..., 497

Let above are n numbers and $a_n = 497$

Here first term, $a = 203$

Common difference $d = 7$

Now, $a_n = 497$

$\Rightarrow \quad a + (n-1)d = 497 \Rightarrow 203 + 7(n-1) = 497$

$\Rightarrow \quad 7(n-1) = 294 \Rightarrow (n-1) = \frac{294}{7} = 42$

$\Rightarrow \quad n = 43$

$\therefore$ There are 43 natural numbers between 200 and 500 divisible by 7.

Question 103.

How many two-digit numbers are divisible by 7?

Solution:

Two digit numbers which are divisible by 7 are 14, 21, 28, ..., 98.

Here first term, $a = 14$; common difference $d = 7$

Let $a_n = 98 \Rightarrow a + (n-1)d = 98$

$\Rightarrow \quad 14 + 7(n-1) = 98 \Rightarrow 7(n-1) = 84$

$\Rightarrow \quad n - 1 = 12 \Rightarrow n = 13.$

Hence, there are 13 two digit numbers which are divisible by 7.

Question 104.

If $\frac{1}{x+2}, \frac{1}{x+3}$ and $\frac{1}{x+5}$ are in A.P., find the value of x.

Solution:

$\because \frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P. We know that

$\Rightarrow \quad \frac{2}{x+3} = \frac{1}{x+2} + \frac{1}{x+5} \Rightarrow \frac{2}{x+3} = \frac{(x+5)+(x+2)}{(x+2)(x+5)}$

$\Rightarrow \quad 2(x+2)(x+5) = (2x+7)(x+3)$

$\Rightarrow \quad 2(x^2 + 7x + 10) = 2x^2 + 13x + 21$

$\Rightarrow \quad 2x^2 + 14x + 20 = 2x^2 + 13x + 21$

$\therefore \quad x = 1$

Short Answer Type Questions II [3 Marks]

Question 105.

Find the value of the middle term of the following AP. -6,-2,2,…, 58

Solution:

Given A.P. is – 6, – 2, 2, ... 58

Here, $a = -6,$

$d = -2 + 6 = 4$

and last term $a_n = 58$

$\Rightarrow \quad a + (n-1)d = 58 \Rightarrow -6 + (n-1)4 = 58$

$\Rightarrow \quad (n-1)4 = 64 \Rightarrow n - 1 = 16$

$\Rightarrow \quad n = 17$

$\therefore$ Since number of terms is odd, so only one middle term.

For middle term, $\left(\frac{17+1}{2}\right)^{th} = \left(\frac{18}{2}\right)^{th} = 9\text{th term}$

$\therefore$ 9th term is the middle term.

So, $a_9 = a + 8d = -6 + 8 \times 4 = 26$

Question 106.

Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

Solution:

Let a be the first term and d be the common difference

Given $a_4 = 18$

$a + 3d = 18$ ...(i)

and also given $a_{15} - a_9 = 30$

$a + 14d - (a + 8d) = 30$

$\Rightarrow (15 - 9)d = 30 \Rightarrow 6d = 30 \Rightarrow d = 5$

Putting the value of d in (i), we have

$a + 3d = 18$

$\Rightarrow a + 3 \times 5 = 18 \Rightarrow a + 15 = 18 \Rightarrow a = 3$

$\therefore$ Required AP is 3, 8, 13, ...

Question 107.

Find an AP ,whose fourth team is 9 and the sum of its sixth term and thirteenth term is 40.

Solution:

Given: $a_4 = 9$

$\Rightarrow a_4 = a + (4-1)d \Rightarrow 9 = a + 3d$ ...(i)

and given $a_6 + a_{13} = 40$

$\Rightarrow a + (6-1)d + a + (13-1)d = 40 \Rightarrow a + 5d + a + 12d = 40$

$\Rightarrow 2a + 17d = 40$ ...(ii)

From (i) and (ii), we have

$2(9 - 3d) + 17d = 40 \Rightarrow 18 - 6d + 17d = 40$

$\Rightarrow 11d = 22 \Rightarrow d = 2$

$\therefore a = 9 - 3 \times 2 = 9 - 6 = 3$

$\therefore a = 3$

A.P. $a, a + d, a + 2d, ..., 3, 5, 7, ...$

Question 108.

Find the sum of first-n-terms of an A.P. whose nth term is 5n – 1. hence find the sum of first 20 terms.

Solution:

Given: $a_n = 5n - 1$

$a_1 = 4; d = 5 = a_2 - a_1 = 9 - 4$

$\therefore a_2 = 5(2) - 1 = 9$

Now, sum of first 'n' terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 4 + 5(n-1)] = \frac{n}{2}(8 + 5n - 5) = \frac{n(5n+3)}{2}$

Now, sum of first 20 terms, $S_{20} = \frac{20(5\times20+3)}{2} = 10 \times 103 = 1030$

Question 109.

Find the sum of all odd integers between 1 and 100, which are divisible by 3.

Solution:

Given: A.P. is 3, 9, 15, 21, ..., 99.

Here, $a = 3; d = 6; a_n = 99$

Now, $a_n = 99$

$\Rightarrow a + (n-1)d = 99 \Rightarrow 3 + 6(n-1) = 99$

$\Rightarrow 6(n-1) = 96 \Rightarrow n - 1 = 16 \Rightarrow n = 17$

Now, sum of 17 terms, $S_{17} = \frac{17}{2}(3 + 99)$ $\left[\because S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= \frac{17}{2} \times 102 = 17 \times 51 = 867$

$\therefore$ Sum of all odd integers between 1 and 100, divisible by 3 is 867.

Long Answer Type Questions [4 Marks]

Question 110.

If the sum of first 4 terms of an AP is 40 and that of first 14 terms is 280, find the sum of its first n terms.

Solution:

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_4 = \frac{4}{2}[2a + (4-1)d] = 40$$

$\Rightarrow$ $2[2a + 3d] = 40 \Rightarrow 2a + 3d = 20$ ...(i)

$$S_{14} = 280$$

and $\frac{14}{2}[2a + (14-1)d] = 280$

$\Rightarrow$ $7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ ...(ii)

Subtracting (i) from (ii), we get

$$10d = 20 \Rightarrow d = 2$$

Putting $d = 2$ in equation (i), we get

$$2a + 3 \times 2 = 20$$

$\Rightarrow$ $2a + 6 = 20 \Rightarrow 2a = 14 \Rightarrow a = 7 = \text{first term}$

$\therefore$ Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[2 \times 7 + (n-1)2] = \frac{n}{2}[14 + 2n - 2]$$

$$= \frac{n}{2}(2n + 12) = n(n + 6)$$

Question 111.

Find the sum of the first 30 positive integers divisible by 6.

Solution:

List of first 30 positive integers divisible by 6 are 6, 12, 18, ...

Here, $n = 30, a = 6, d = 6$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ Sum of 30 terms, $S_{30} = \frac{30}{2}[2 \times 6 + (30-1)6]$

$$= 15[12 + 29 \times 6] = 15[12 + 174] = 15[186] = 2790$$

$\therefore$ Sum of first 30 positive integers, divisible by 6 is 2790.

Question 112.

The first and the last terms of an AP are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum?

Solution:

Here, $a = 8, a_n = 350, d = 9$

Now, $a_n = 350$

$\Rightarrow$ $a + (n-1)d = 350 \Rightarrow 8 + (n-1)9 = 350$

$\Rightarrow$ $(n-1)9 = 350 - 8 \Rightarrow (n-1)9 = 342$

$\Rightarrow$ $n - 1 = \frac{342}{9} \Rightarrow n - 1 = 38 \Rightarrow n = 38 + 1$

$\therefore$ $n = 39$

Now, $S_n = \frac{n}{2}(a + a_n)$, we get

$\because$ Sum of 39 terms, $S_{39} = \frac{39}{2}(8 + 350) = \frac{39}{2} \times 358 = 6981$

Question 113.

How many multiples of 4 lie between 10 and 250? Also find thier sum.

Solution:

Required A.P. is 12, 16, 20, ..., 240, 244, 248

Here, $a = 12; d = 4; a_n = 248$ = last term

Then, $a_n = 248$

$\Rightarrow a + (n-1)d = 248$

$\Rightarrow 12 + 4(n-1) = 248 \Rightarrow 4(n-1) = 236$

$\Rightarrow n - 1 = 59 \Rightarrow n = 60$

Hence, there are 60 numbers which are multiples of 4 lie between 10 and 250.

Now, sum of these multiples, $S_{60} = \frac{60}{2}(12 + 248)$ $\left[\because S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= 30 \times 260 = 7800$

Question 114.

In an AP, if the 6th and 13th terms are 35 and 70 respectively, find the sum of its first 20 terms.

Solution:

Given that, $a_6 = 35$

$\Rightarrow a + 5d = 35$ ...(*i*)

and also $a_{13} = 70$

$\Rightarrow a + 12d = 70$ ...(*ii*)

On solving the above equations, we get

$a = 10; d = 5$

Now, sum of first 20 terms, $S_{20} = \frac{20}{2}[2 \times 10 + 19 \times 5]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 10 \times (20 + 95) = 10 \times 115 = 1150$

Question 115.

In an AP, if the sum of its 4th and 10th terms is 40, and the sum of its 8th and 16th terms is 70, then find the sum of its first twenty terms.

Solution:

Given: $a_4 + a_{10} = 40$, where an $= a_n = a + (n-1)d$

$\Rightarrow a + 3d + a + 9d = 40$

$\Rightarrow 2a + 12d = 40 \Rightarrow a + 6d = 20$ ...(*i*)

Also, given $a_8 + a_{16} = 70$

$\Rightarrow a + 7d + a + 15d = 70$

$\Rightarrow 2a + 22d = 70 \Rightarrow a + 11d = 35$ ...(*ii*)

On solving the above equations, we get

$a = 2$ and $d = 3$

Now, sum of first 20 terms, $S_{20} = \frac{20}{2}[2 \times 2 + 19 \times 3]$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 10(4 + 57) = 10 \times 61 = 610$

Question 116.

In an A.P., if the sum of 4th and the 8th terms is 70 and its 15th term is 80, then find the sum of its first 25 terms.

Solution:

Accordting to question, $a_4 + a_8 = 70$, where $a_n = a + (n-1)d$

$\Rightarrow a + 3d + a + 7d = 70$

$\Rightarrow 2a + 10d = 70 \Rightarrow a + 5d = 35$ ...(*i*)

and also given $a_{15} = 80$

$\Rightarrow a + 14d = 80$ ...(*ii*)

On solving the equations (*i*) and (*ii*), we get $a = 10; d = 5$

Now, sum of first 25 terms, $S_{25} = \frac{25}{2}(2 \times 10 + 24 \times 5) = \frac{25}{2} \times 140 = 1750$

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Very Short Answer Type Questions [1 Mark]

Question 117.

If the sum of first p terms of an AP is ap2 + bp, find its common difference.

Solution:

Given that, $S_p = ap^2 + bp$

$S_1 = a + b = T_1 = \text{First term}$ [put $p = 1$]

$S_2 = 4a + 2b$ [put $p = 2$]

$S_3 = 9a + 3b$ [put $p = 3$]

$\therefore$ 2nd term, $T_2 = S_2 - S_1 = 4a + 2b - a - b = 3a + b$

$\therefore$ 3rd term, $T_3 = S_3 - S_2 = 9a + 3b - 4a - 2b = 5a + b$

Common difference $= T_3 - T_2 = 5a - b + 3a - b = 2a$

Alternative:

Common difference $(d) = 2a$ [$\because$ Twice the coefficient of p^2 in Sp of an A.P. is the common difference]

Question 118.

If the sum of the first q terms of an AP is 2q + 3q2, what is its common difference?

Solution:

Given that, $S_q = 2q + 3q^2$

$S_1 = 2 + 3 = 5 = T_1 = \text{First term}$ [put $q = 1$]

$S_2 = 4 + 3(4) = 16$ [put $q = 2$]

$S_3 = 6 + 3(9) = 33$ [put $q = 3$]

$\therefore$ 2nd term, $T_2 = S_2 - S_1 = 16 - 5 = 11$

$\therefore$ 3rd term, $T_3 = S_3 - S_2 = 33 - 16 = 17$

Common difference $= T_3 - T_2 = 17 - 11 = 6$

Question 119.

If the sum of first m terms of an AP is 2m2 + 3m, then what is its second term?

Solution:

Given that, $S_m = 2m^2 + 3m$

Here, $a = S_1 = 2 \times 1^2 + 3 \times 1 = 5$ [$\because$ Put $m = 1$]

$d = 2 \times 2 = 4$ [$\because$ Twice the coefficient of m^2 in S_m of an A.P. is the common difference]

Now, second term, $a_2 = a + d = 5 + 4 = 9$

Short Answer Type Questions I [2 Marks]

Question 120.

In an AP, the first term is 2, the last term is 29 and sum of n terms is 155. Find the common difference of the AP.

Solution:

In the given AP, $a = 2, l = 29$

$S_n = 155.$

$\therefore$ $155 = \frac{n}{2}(2 + 29)$ $\left[\because S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow$ $310 = 31n \Rightarrow n = 10$

Now, last term, $a_{10} = l = 29$

$\Rightarrow$ $29 = a + 9d \Rightarrow 27 = 9d$ [$\because a_n = a + (n-1)d$]

$\Rightarrow$ $d = 3$

$\therefore$ Common difference $= 3$

Question 121.

Find the common difference of an AP whose first term is 4, the last term is 49 and the sum of all its terms is 265.

Solution:

In the given AP, $a = 4, l = 49$ and $S_n = 265$.

$\therefore \quad 265 = \frac{n}{2}(4 + 49) \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$

$\Rightarrow \quad 530 = 53n \Rightarrow n = 10$

$\therefore$ Then, we have $\quad l = a_{10} = a + 9d$

$\Rightarrow \quad 49 = 4 + 9d$

$\Rightarrow \quad 9d = 45 \Rightarrow d = 5$

$\therefore$ Common difference $= 5$

Question 122.

In an AP, the first term is – 4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

Solution:

In the given AP, $a = -4, l = 29, S_n = 150$.

$\therefore \quad 150 = \frac{n}{2}(-4 + 29) \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$

$\Rightarrow \quad 300 = 25n \Rightarrow n = 12$

$\therefore$ Then, $\quad l = a_{12} = 29 = -4 + 11d \Rightarrow 11d = 33 \Rightarrow d = 3$

$\therefore$ Common difference $= 3$.

Short Answer Type Questions II [3 Marks]

Question 123.

In an AP, the sum of first ten terms is – 150 and the sum of its next ten terms is – 550. Find the AP.

Solution:

Given $\quad S_{10} = -150$

$\Rightarrow \quad \frac{10}{2}(2a + 9d) = -150 \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\Rightarrow \quad 2a + 9d = -30 \qquad ...(i)$

and also $\quad S_{20} - S_{10} = -550 \Rightarrow S_{20} = -550 + (-150)$

$\Rightarrow \quad S_{20} = -700 \Rightarrow \frac{20}{2}(2a + 19d) = -700$

$\Rightarrow \quad 2a + 19d = -70 \qquad ...(ii)$

On solving eqn(s) (*i*) and (*ii*), we get

$d = -4$ and $a = 3$

$\therefore$ Required AP is 3, – 1, – 5, – 9, ...

Question 124.

In an AP, the sum of first ten terms is – 80 and the sum of its next ten terms is – 280. Find the AP.

Solution:

Given that $\quad S_{10} = -80 \Rightarrow \frac{10}{2}(2a + 9d) = -80$ as $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 2a + 9d = -16 \qquad ...(i)$

and also $\quad S_{20} - S_{10} = -280 \Rightarrow S_{20} + 80 = -280$

$\Rightarrow \quad S_{20} = -360 \Rightarrow \frac{20}{2}(2a + 19d) = -360$

$\Rightarrow \quad 2a + 19d = -36 \qquad ...(ii)$

On solving eqn(s) (*i*) and (*ii*), we get

$d = -2$ and $a = 1$

$\therefore$ Required AP is 1, – 1, – 3, – 5, ...

Question 125.

The sum of the first sixteen terms of an AP is 112 and the sum of its next fourteen terms is

518. Find the AP.

Solution:

Given that $S_{16} = 112 \Rightarrow \frac{16}{2}(2a + 15d) = 112$ as $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $2a + 15d = 14$ ...(i)

and also $S_{30} - S_{16} = 518 \Rightarrow S_{30} - 112 = 518$

$\Rightarrow$ $S_{30} = 630 \Rightarrow \frac{30}{2}(2a + 29d) = 630 \Rightarrow 2a + 29d = 42$ ...(ii)

On solving eqn(s) (i) and (ii), we get

$d = 2$ and $a = -8$

$\therefore$ Required AP is $-8, -6, -4, \ldots$